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Interior Solution Gravity Experiment

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Abstract. —
For practical and historical reasons, most of what we know about gravity is based on observations made or experiments conducted *beyond the surfaces* of dominant massive bodies. Although the *force* of gravity inside a massive body can sometimes be measured, it remains to demonstrate the *motion* that would be caused by that force through the body's center. Since the idea of doing so has often been discussed as a thought experiment, we here look into the possibility of turning this into a real experiment. Feasibility is established by considering examples of similar experiments whose techniques could be utilized for the present one.

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1. – Introduction

Often found in undergraduate physics texts [1, 2, 3, 4] is the following problem, discussed in terms of Newtonian gravity: A test object is dropped into an evacuated hole spanning a diameter of an otherwise uniformly dense spherical mass. One of the reasons this problem is so common is that the answer, the predicted equation of motion of the test object, is yet another instance of *simple harmonic motion*. What is rarely pointed out, however, is that we presently lack direct empirical evidence to verify the theoretical prediction. Confidence in the prediction is primarily based on the success of Newton's theory for phenomena that test the *exterior* solution. Extrapolating Newton's law to the interior is of course a worthwhile mathematical excercise. But a theoretical extrapolation is of lesser value than an empirical fact.

Essentially the same prediction follows from general relativity. [5, 6, 7, 8] Since there is no obvious reason to doubt the predicted simple harmonic motion, in this context too, the impression is sometimes given that it is a physical fact. A noteworthy example is found in John A. Wheeler's book, *A Journey Into Gravity and Spacetime*, in which he refers to the phenomenon as "boomeranging." Wheeler devotes a whole (10-page) chapter to the subject because, as he writes, "Few examples of gravity at work are easier to understand in Newtonian terms than boomeranging. Nor do I know any easier doorway to Einstein's concept of gravity as manifestation of spacetime curvature." [9] But nowhere in Wheeler's book is there any discussion of *empirical evidence* for "boomeranging." No doubt, Newton, Einstein and Wheeler would all have been delighted to see the simple harmonic motion demonstrated in a laboratory experiment.

Since the predicted effect has never been observed even as an approximation, our initial goal should be modest: to roughly demonstrate the principle first and perhaps later concern ourselves with high precision. After laying out a basic strategy for doing the experiment, this essay concludes with a few additional remarks concerning motivation.

2. – Feasibility

What kind of apparatus do we need? In the 1960s–1970s a few proposals to measure Newton's constant, G, involved through-the-center oscillations. Y. T. Chen discusses these ideas in his 1988 review paper on G measurements. [10] Each example in this particular group of proposals was intended for space-borne *satellite* laboratories. The original motivation of these ideas was to devise ways to improve the accuracy of our knowledge of G by timing the oscillation period of the simple harmonic motion. Though having some advantages over Earth-based G measurements, they also had drawbacks which ultimately prohibited them from ever being carried out.

What distinguishes these proposals from experiments that have actually been carried out in Earth-based laboratories is that the test objects were to be allowed to *fall freely* back and forth between extremities inside a source mass the whole time. Whereas G measurements conducted on Earth typically involve *restricting* the test mass's movement and measuring the *force* needed to do so. The most common, and historically original, method for doing this is to use a torsion balance, where a fiber provides a predetermined resistance to rotation. Torsion balances have also been used to test Einstein's Equivalence Principle (e.g., Gundlach, et al [11]). Another distinguishing characteristic of Earth-based G measurements and Equivalence Principle tests is that the test masses typically remain *outside* the larger source masses. Since movement of the test masses is restricted to a small range of motion, these tests can be characterized as *static* measurements. Torsion balance experiments in which the test mass is *inside* the source mass have also been performed. (For example, Spero, et al [12] and Hoskins, et al [13].) These latter experiments were tests of the inverse square law.

All three of these types of experiments – G measurements, Equivalence Principle and inverse-square law tests – however, are *static* measurements in the sense that the test masses were not free to move beyond a small distance compared to the size of the source mass. The key innovation in the present proposal is that we want to see an object fall radially as long as it will; we want to eliminate (ideally) or minimize (practically) any obstacle to the radial free-fall trajectory. Space-based experiments would clearly be the optimal way to achieve this. But a reasonably close approximation can be achieved with a modified Cavendish balance in an Earth-based laboratory.

As implied above, the key is to design a suspension system which, instead of providing a restoring force that prevents the test masses from moving very far, allows unrestricted or nearly unrestricted movement. Two available possibilities are fluid suspensions and magnetic suspensions (or a combination of these). In 1976 Faller and Koldewyn succeeded in using a magnetic suspension system to get a G measurement. [14, 15] The experiment's accuracy was not an improvement over that gotten by other methods, but was within 1.5% of the standard value.

In the apparatus Cavendish used for his original G measurement the torsion arm and test masses were isolated from the source masses by a wooden box. In Faller and Kold-

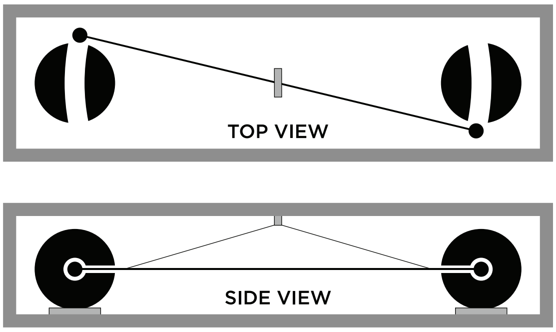

Fig. 1. – Schematic of modified Cavendish balance. Since the idea is to demonstrate the simple harmonic motion only as a first approximation, deviation due to the slight arc in the trajectory is inconsequential.

ewyn's experiment the arm was isolated from the source masses by a vacuum chamber. The modified design requires that there be no such isolation, as the arm needs to swing freely through the center of the source masses. (See Figure 1.) Given the modest goal of the present proposal, it is reasonable to expect that the technology used by Faller and Koldewyn could be adapted to demonstrate the oscillation prediction. Moreover, it seems reasonable to expect that advances in technology (e.g., better magnets, better electronics, etc.) since 1976 would make the experiment quite doable for an institution grade physics laboratory.

3. – Motivation: Completeness and Aesthetics

One hardly needs to mention the many successes of Newtonian gravity. By success we mean, of course, that empirical observations match the theoretical predictions. Einsteinian gravity is even more successful. The purpose of many contemporary gravity experiments is to detect physical manifestations of the differences between Newton's and Einstein's theories. In every case Einstein's theory has proven to be more accurate. This is impressive. Given the level of thoroughness and sophistication in gravity experimentation these days one may be taken aback to realize that Newton's and Einstein's theories *both remain untested* with regard to the problem discussed above. The simple harmonic motion prediction is so common and so obvious that we have come to take it for granted. Wouldn't it be more satisfactory if, when discussing the *prediction*, we could at the same time cite the *physical evidence*?

The Newtonian explanation for the predicted harmonic motion is, of course, that a massive sphere produces a force (or potential) of gravitational attraction. The corresponding general relativistic explanation is that the curvature of spacetime causes the motion. Specifically, the predicted effect is due to the slowing of clock rates toward the center of the sphere. A physical demonstration of the effect would thus indirectly, though convincingly, support general relativity's prediction that the rate of a clock at the body's center is a local minimum—a prediction that has otherwise not yet been confirmed.

In summary, if R represents the surface of a spherical mass, our empirical knowledge of how things move because of the mass within R is essentially confined to the region, $r \gtrsim R$. The region $0 \leq r \lesssim R$ is a rather basic and a rather large gap. It is clearly the most *ponderable* part of the domain. Why not fill this gap?

One of the distinctive features of the kind of experiment proposed above is that its result is, in principle, independent of *size*. The satellite versions mentioned by Chen were thus referred to as "clock mode" experiments. The determining factor in the oscillation period is the *density* of the source mass. If the source mass is made of lead (density, $\rho \approx 11,000\,\mathrm{kg}\,/\,\mathrm{m}^3$) the oscillation period is about one hour. I'd guess that many students and physicists would be fascinated to observe for an hour, to watch the oscillation take place, knowing that the *mass* of the larger body is the essential thing making it happen. In my opinion this would be a *beautiful* sight. Beautiful for completing the domain, $0 \leq r \lesssim R$, and beautiful simply to see what no human being has seen before.

REFERENCES

[1] Halliday D., Resnick R. and Walker J., *Fundamentals of Physics* (Wiley, New York) 1993, p. 419.

[2] Valens E. G., *The Attractive Universe: Gravity and the Shape of Space* (World, Cleveland) 1969, pp. 145-149.

[3] Tipler P. A., *Physics* (Worth, New York) 1982, p. 362.

[4] French A. P., *Newtonian Mechanics* (Norton, New York) 1971, pp. 144, 483-484.

[5] Misner C. W., Thorne K. and Wheeler J. A., *Gravitation* (W. H. Freeman, San Francisco) 1973, pp. 37-39.

[6] Tangherlini F. R., An Introduction to the General Theory of Relativity, *Nuovo Cimento Supplement*, **20** (1966) 66.

[7] Epstein L. C., *Relativity Visualized* (Insight, San Francisco) 1988, pp. 157-158.

[8] Taylor N. W., Note on the Harmonic Oscillator in General Relativity, *Australian Journal of Mathematics*, **2** (1961) 206.

[9] Wheeler J. A., *A Journey Into Gravity and Spacetime* (Scientific American Library, New York) 1990, pp. 55-65.

[10] Chen Y. T., *The Measurement of the Gravitational Constant*, in *International Symposium on Experimental Gravitational Physics*, edited by Michelson P. F., En-ke H. and Pizzella G. (World Scientific, Singapore) 1988, pp. 90-109.

[11] Gundlach J. H., Smith G. L., Adelberger E. G., Heckel B. R. and Swanson H. E., Short-Range Test of the Equivalence Principle, *Physical Review Letters*, **78** (1997) 2523-2526.

[12] Spero R. E., Hoskins J. K., Newman R., Pellam J. and Schultz J., Test of the Gravitational Inverse-Square Law at Laboratory Distances, *Physical Review Letters*, **44** (1980) 1645-1648.

[13] Hoskins J. K., Newman R. D., Spero R. E. and Schultz J., Experimental tests of the gravitational inverse-square law for mass separations from 2 to 105 cm, *Physical Review D*, **32** (1985) 3084-3095.

[14] Koldewyn W. A., *A New Method for Measuring the Newtonian Gravitational Constant, G*, PhD thesis (Wesleyan University) 1976.

[15] Faller J. E., *A Prototype Measurement of the Newtonian Gravitational Constant Using an Active Magnetic Suspension Torsion Fiber*, in *Proceedings of the 1983 International School and Symposium on Precision Measurement and Gravity Experiment*, edited by Wei-Tou N. (National Tsing Hua University, Hsinchu, Taiwan) 1983, pp. 541-556.